

# On the range of validity of the QCD-improved parton model

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February 1, 2008

## Abstract

Based on the world DIS data we extract the experimental  $F_2^p - F_2^n$  as a function of Bjorken- $x$  and photon virtuality  $Q^2$ , using two different methods. Both methods lead to identical results. We find that the standard PDFs fail to describe the experimental data below  $Q^2 < 7$  GeV<sup>2</sup>, which is much higher than for  $F_2^p$  and  $F_2^d$  separately. The difference between PDFs and the experimental data cannot be understood as due to nuclear effects in the deuteron, and evidently suggests substantial nonsinglet higher-twist effects. The trend of the experimental data is approximately explained by a recent two-component model of the nucleon structure functions and suggests strong  $Q^2$ -dependence of the Gottfried Sum, in disagreement with the parton model interpretation. The large negative higher-twist effects can explain the difference between the value of the Gottfried integral obtained recently by the E866 Drell-Yan experiment at Fermilab and an older NMC result.

The QCD-improved parton model (IPM) applies at large virtuality of the probe. However, the exact range of its applicability is not obvious. Generally a good description of  $F_2^p$  and  $F_2^d$  within IPM can be obtained already above  $Q^2 \sim 1$ -2 GeV<sup>2</sup> [1, 2, 3].

In the last decade the precision of experimental data on structure functions became so good that one can try to obtain the information on effects beyond the leading twists [4, 5].

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A recent lattice QCD calculation has found an unexpectedly large twist-4 contribution to the structure functions [6]. This is rather difficult to reconcile in the light of the success of the improved parton model which is known to work empirically down to  $Q^2 \sim 1\text{-}2 \text{ GeV}^2$ .

In a recent work[7] we have shown that both the proton and deuteron structure functions can be well explained in a broad range of  $x$  and  $Q^2$  also in a two-component model of the structure function

$$F_2^{p/n}(x, Q^2) = F_2^{p/n, had}(x, Q^2) + F_2^{p/n, part}(x, Q^2) , \quad (1)$$

which gives better agreement with data below  $Q^2 \sim 3 \text{ GeV}^2$ . Our model fulfills  $F_2^{p/n}(x, Q^2) \rightarrow 0$  when  $Q^2 \rightarrow 0$ , as required by current conservation. While  $F_2^{p/n, had}(x, Q^2) \rightarrow 0$  by construction, the vanishing of  $F_2^{p/n, part}(x, Q^2)$  is achieved as in the low- $x$  Badełek-Kwieciński model [8]

$$F_2^{p/n, part}(x, Q^2) = \frac{Q^2}{Q^2 + Q_0^2} \cdot F_2^{p/n, asymp}(\bar{x}, \bar{Q}^2) , \quad (2)$$

where  $F_2^{p/n, asymp}$  is the standard parton-model structure function and  $\bar{x}$ ,  $\bar{Q}^2$  are defined as in [8]. This simple phenomenological form insures a correct  $Q^2 \rightarrow 0$  limit and is justified by the dispersion method [8]. For not too small  $x$ , i.e. in the region we are interested in here, the modification of the structure function arguments  $x \rightarrow \bar{x}$ ,  $Q^2 \rightarrow \bar{Q}^2$  is not needed [7].

Except for a better description of the  $F_2$  data in the low- $Q^2$  region the model from [7] has some interesting consequences. The higher-twist contribution to the structure function can be defined by

$$\delta^{HT} F_2^{p/n}(x, Q^2) \equiv F_2^{p/n}(x, Q^2) - F_2^{p/n, asymp}(x, Q^2) . \quad (3)$$

In the model for  $F_2^p$ ,  $F_2^n$  proposed in [7] there is a significant cancellation between positive (VDM-type) and negative (due to a modification of the partonic component) higher twist contributions

$$\begin{aligned} \delta^{HT} F_2^{p/n}(x, Q^2) &= \\ &= F_2^{p/n, had}(x, Q^2) + \frac{Q^2}{Q^2 + Q_0^2} \cdot F_2^{p/n, asymp}(\bar{x}, \bar{Q}^2) - F_2^{p/n, asymp}(x, Q^2) \approx \\ &\approx F_2^{p/n, had}(x, Q^2) - \frac{Q_0^2}{Q^2 + Q_0^2} \cdot F_2^{p/n, asymp}(x, Q^2) \end{aligned} \quad (4)$$

in the range of small and intermediate  $x$ .

Neglecting a small  $\rho$ - $\omega$  nondiagonal term due to the exchange of a Regge trajectory  $a_2$

$$\delta^{(\rho-\omega)} F_2^{p/n}(x, Q^2) = \pm \frac{Q^2}{\pi} \cdot C_{\rho,\omega}^{a_2} \cdot \frac{m_\rho^2}{(Q^2 + m_\rho^2)} \cdot \frac{m_\omega^2}{(Q^2 + m_\omega^2)} \cdot \Omega_{\rho,\omega}(x, Q^2), \quad (5)$$

we have

$$\begin{aligned} F_2^{p, had}(x, Q^2) &= F_2^{n, had}(x, Q^2) \\ &\equiv F_2^{had}(x, Q^2) = \frac{Q^2}{\pi} \sum_V C_V \frac{M_V^4}{(Q^2 + M_V^2)^2} \cdot \Omega_V(x, Q^2), \end{aligned} \quad (6)$$

where  $C_V$  is a normalization factor which can be related to the electromagnetic  $V \rightarrow e^+e^-$  decay width and  $\Omega(x, Q^2)$  is a form factor which phenomenologically accounts for the finite lifetime of the hadronic fluctuation of the photon [7]. Therefore

$$\begin{aligned} D_G(x, Q^2) &\equiv F_2^p(x, Q^2) - F_2^n(x, Q^2) = \\ &= \frac{Q^2}{Q^2 + Q_0^2} \cdot \{F_2^{p, asym}(x, Q^2) - F_2^{n, asym}(x, Q^2)\} \end{aligned} \quad (7)$$

for the range of Bjorken- $x$  we are interested in. Unlike the case of the structure functions there is no cancellation of hadronic and partonic higher twists. A strong  $Q^2$ -dependence of  $D_G$  is obvious. The factor in front of the r.h.s. of Eq.(7) assures also the vanishing of the Gottfried integral for  $Q^2 \rightarrow 0$ . This result is in contrast with the result obtained by Ball and Forte [9] who found a restoration of the Gottfried Sum Rule  $S_G = \int_0^1 \frac{dx}{x} \{F_2^p(x, Q^2) - F_2^n(x, Q^2)\} \rightarrow \frac{1}{3}$  when  $Q^2 \rightarrow 0$ .

In order to verify the anticipated effect discussed above let us come to the status of experimental data on  $F_2^p - F_2^n$ . Because  $F_2^n$  is not a directly measurable quantity one is forced to use rather deuteron data. The simplest approximation, neglecting all nuclear effects in the deuteron, would be to replace  $D_G \equiv F_2^p - F_2^n$  by  $F_2^p - (2F_2^d - F_2^p)$ . This direct method is, however, not very useful as far as the existing data are concerned. There the errors (both systematical and statistical) for  $F_2^p$  and  $F_2^d$  are independent and the resulting errors for  $D_G$  are too large to reveal the anticipated  $Q^2$ -dependence. Instead of the direct subtraction of  $F_2^d$  and  $F_2^p$  one can use a method proposed by NMC at CERN [10]. In the following two methods will be used to extract  $D_G$ :

- (A):  $F_2^p(x, Q^2) - F_2^n(x, Q^2) = F_2^p(x, Q^2) \cdot \left(1 - \frac{F_2^n(x, Q^2)}{F_2^p(x, Q^2)}\right)$  ,
- (B):  $F_2^p(x, Q^2) - F_2^n(x, Q^2) = 2F_2^d(x, Q^2) \cdot \frac{1 - \frac{F_2^n(x, Q^2)}{F_2^p(x, Q^2)}}{1 + \frac{F_2^n(x, Q^2)}{F_2^p(x, Q^2)}}$  ,

where  $F_2^n/F_2^p$  is extracted from the measured  $F_2^d/F_2^p$  data. The latter was already used by NMC in the evaluation of the Gottfried integral in an unpublished analysis of the  $Q^2$  dependence of the NMC data [11]. The two methods above are of course equivalent when nuclear effects are neglected. They are equivalent also in a less obvious case when nuclear effects are taken into account and  $F_2^n/F_2^p$  is replaced by  $2F_2^d/F_2^p - 1$  (this will be assumed hereafter). It can be shown that then both methods include nuclear effects<sup>1</sup> in exactly the same way, identical as the difference  $F_2^p - (2F_2^d - F_2^p)$ .

Both methods require knowledge of the ratio  $F_2^n/F_2^p$  for the same  $x$  and  $Q^2$  points as the structure functions  $F_2^p$  or  $F_2^d$  which is not the case for existing experimental data. For this reason use of a parametrization for  $F_2^n/F_2^p$  appears necessary. In our analysis we have used a parametrization from [5]. In Fig.1 (top panel) we compare this parametrization with precise  $F_2^d/F_2^p$  data from NMC [12]. The dashed lines indicate a 2 % uncertainty. Below we shall quantify the quality of the parametrization and justify the choice of its uncertainty band. In the bottom-left panel we present a percentage of points above (solid line) and below (dashed line) a trial (rescaled) parametrization for the ratio, as a function of the relative deviation from the original parametrization. The result clearly indicates that statistically the same amount of points lies above and below the nominal curve. In the bottom-right panel we show the fraction of NMC data points in a band of uncertainty as a function of the band width (dashed line). About 3/4 of the experimental points remain in the 2 % band of uncertainty. Because the functional form of the ratio is not known, and some fluctuations of the ratio are not excluded a priori, we have also performed the following analysis. We assign to each experimental point a Gaussian distribution of probability, centered on the experimental point, with a standard deviation equal to the experimental error. We then determine the fraction of the area that lies below such a probability distribution within the parametrization uncertainty. We show this quantity averaged over 260 NMC experimental points [12] in the bottom-right panel by the solid line marked by  $\langle P \rangle$ . About 60 % of

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<sup>1</sup>the latter will be marked by a star in Fig.2 and 3.

so-defined probability remain in the 2 % band of uncertainty.<sup>2</sup> This analysis assumes inherently independence of successive experimental points and allows fluctuations around the smooth parametrization. Because we do not expect neither a full independence of neighbouring experimental points nor sharp fluctuations of the ratio, the 2 % uncertainty band seems to be rather an upper limit for the uncertainty on  $F_2^n/F_2^p$ . Taking into account uncertainty of  $F_2^n/F_2^p$  at its upper limit should allow us to prove our thesis about the IPM breaking with a bigger degree of reliability. We have checked that in the interesting kinematical range the used parametrization is consistent within experimental uncertainties with the E665 [13] data for  $F_2^d/F_2^p$ .

The world data on  $F_2^p$  and  $F_2^d$  from the compilation [14] was taken in the present analysis.

In Fig.2 we show experimental data for  $D_G(x)$  obtained with the two methods for three different values of photon virtuality  $Q^2 = 1.1, 3.4, 7.0$  GeV<sup>2</sup> as a function of Bjorken- $x$ . The error bars take into account the inherent uncertainties of  $F_2^p$  (method A) or  $F_2^d$  (method B) as well as the uncertainty of  $F_2^n/F_2^p$ . The two bands below each distribution separately show the errors due to the mentioned above 2% uncertainty in  $F_2^n/F_2^p$  and due to the uncertainty of  $F_2^p$  (lhs) or  $F_2^d$  (rhs). As seen from the figure the error of  $D_G$  due to the uncertainty of the  $F_2^n/F_2^p$  ratio is substantially bigger than the one due to the uncertainty of either  $F_2^p$  or  $F_2^d$ . Within experimental errors the two methods lead to identical results despite the different data sets used. We have also checked that the result obtained by a direct subtraction of the deuteron and proton structure functions leads to a consistent result although the scatter of the data is much larger. Therefore the data obtained in this way can be reliably used to study the  $Q^2$  dependence of  $F_2^p - F_2^n$  and/or corresponding higher-twist effects.

For comparison we show theoretical results obtained with GRV94 [1] (short-dashed line), MRST [2] (long-dashed line) and a recent CTEQ5 [3] (solid line) NLO DIS-scheme parton distributions.<sup>3</sup> Above  $Q^2 > 7$  GeV<sup>2</sup> the parton model describes the experimental data well. However, the predictions of the improved parton model clearly deviate from the experimental data below  $Q^2 < 7$  GeV<sup>2</sup> for Bjorken  $x$  between 0.2 and 0.45. This deviation is not a random fluctuation. It is present in a broad range of  $Q^2$  as will be shown in the next figure. Similar deviations of the IPM from the experimental data

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<sup>2</sup>Please note that within standard statistical analysis, assuming the Gaussian probability distribution, the required fraction for the standard deviation is 68.2 %

<sup>3</sup>The MRST result is shown only above  $Q^2 > 1.25$  GeV<sup>2</sup> and CTEQ5 above  $Q^2 > 1$  GeV<sup>2</sup>, i.e. in their applicability range.

can also be observed for  $F_2^p$  and  $F_2^d$ , albeit at smaller values of  $Q^2$  [7]. This is consistent with the cancellation of higher-twists for  $F_2^p$  or  $F_2^d$  and with the lack of such a cancellation for  $F_2^p - F_2^n$  as discussed above. None of nuclear effects we know can explain this disagreement.<sup>4</sup> Moreover the agreement of the improved parton model with experimental data for small values of  $x$  is most probably accidental, as shadowing effects which are not included in this calculation, but are present in the data, would enhance the theoretical results by 0.005-0.015, depending on  $(x, Q^2)$  and the model used. This would worsen the agreement of the IPM also for small values of Bjorken  $x < 0.1$ . For the sake of simplicity, we omit nuclear corrections in the present paper. We have estimated them and found that they do not affect our conclusions. For comparison we show the prediction of our model[7] (thick solid line) with  $Q_0^2 = 0.79 \text{ GeV}^2$ , with target mass corrections calculated according to [16]. The target mass effects become important at small  $Q^2$  and larger  $x$ .

In order to better visualize the unexpected within the IPM  $Q^2$ -dependence we present in Fig.3  $D_G$  as a function of  $Q^2$  for some selected values of Bjorken  $x = 0.140, 0.275, 0.350$ . This figure clearly demonstrates a failure of the IPM as well as substantial negative higher-twists for  $F_2^p - F_2^n$ . The trend of the experimental data is completely different to the prediction of the standard IPM. Our model reproduces well the main trend of the experimental data and provides a better description than the standard IPM. However, the agreement of our simple model with experimental data is not complete. The structure function difference  $F_2^p - F_2^n$  is a relatively small quantity and therefore may be sensitive to subtle effects. At low  $Q^2$ , i.e. rather low energy for fixed Bjorken- $x$ , one should worry about a careful treatment of different exclusive channels and their contribution to  $F_2$ . One may also expect some contribution due to the  $\rho - \omega$  nondiagonal VDM term (5). NNLO analysis and careful treatment of nuclear effects could also be valuable.

The effect discussed in the present paper may be very important to understand small differences for light-antiquark flavour asymmetry obtained from different types of experiments. Recently the E866 experiment on Drell-Yan production in proton-proton and proton-deuteron scattering has provided a new, rather precise, information about the nucleon  $\bar{d} - \bar{u}$  asymmetry [17]. Because of the broad range of  $x$  available in this experiment we could estimate

$$\int_0^1 dx [\bar{d}(x) - \bar{u}(x)]_{E866} = 0.09 \pm 0.02 . \quad (8)$$

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<sup>4</sup> No strong  $Q^2$ -dependence is predicted for nuclear effects except of a VDM-type shadowing [15] for  $x < 0.1$ .

This number is smaller than the one deduced from a somewhat older NMC-result [10] on the Gottfried Sum Rule <sup>5</sup>

$$\int_0^1 dx [\bar{d}(x) - \bar{u}(x)]_{NMC} = 0.147 \pm 0.039 . \quad (9)$$

While the average photon virtuality in the Fermilab experiment was fairly large  $\langle Q^2 \rangle \sim 50 \text{ GeV}^2$ , the average photon virtuality in the CERN deep inelastic scattering experiment was relatively low:  $\langle Q^2 \rangle \sim 4 \text{ GeV}^2$ . The latter experiment is in the range of  $Q^2$  where we have observed large negative higher-twist effects (see Figs.2 and 3). This means that the deviation of the Gottfried Sum Rule from its classical value of  $\frac{1}{3}$  [18] observed by NMC [10] is partially due to the  $\bar{d} - \bar{u}$  asymmetry and partially due to the discussed higher-twist effects as follows from Eq.(7). The higher-twist effects would resolve then the discrepancy between (8) and (9).

In summary, we have found substantial deviations from the improved parton model for the difference  $F_2^p - F_2^n$  already at  $Q^2$  as large as  $7 \text{ GeV}^2$ , which is much higher than for  $F_2^p$  and  $F_2^d$  separately. The deviation has been predicted by us in [7]. When combined, these two analyses strongly suggest that the agreement of the improved parton model for  $F_2^p$  and  $F_2^d$  at  $Q^2 < 7 \text{ GeV}^2$  is rather accidental. The present result is in contrast to rather small higher-twist effects estimated in  $F_3^{\nu N}(x, Q^2)$  [19]. The nuclear effects in the deuteron have not been included in the present analysis. However, we estimated their role within simple nuclear models and found that taking into account these effects would only strengthen our conclusions. The results obtained here strongly suggest the  $Q^2$ -dependence of the Gottfried Sum Rule and could resolve a discrepancy between the  $\bar{u} - \bar{d}$  asymmetry obtained in different processes. The breaking of the parton model approach for the reported  $Q^2$  range would have important consequences for different analyses of exclusive reactions concerning the flavour and spin structure of the nucleon where the applicability of the parton model is *conditio sine qua non*.

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<sup>5</sup>We note that both analyses assume isospin symmetry for proton and neutron quark distributions.

**Acknowledgments** We are indebted to Antje Brüll for the discussion of details of the NMC data and Jan Kwieciński for the discussion on the Badelek-Kwieciński model. We are also grateful to Rudolf Tegen for a careful reading of the manuscript and useful comments. This work was partially supported by the German-Polish DLR exchange program, grant number POL-028-98.



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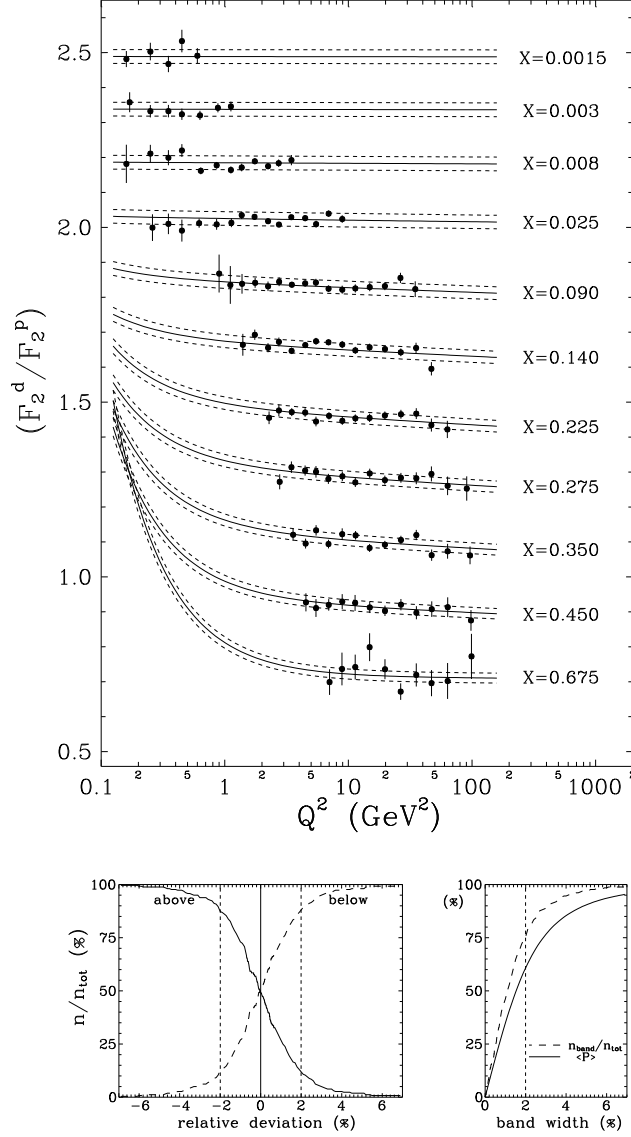


Figure 1: The quality of the  $F_2^n/F_2^p$  ratio parametrization for the NMC  $F_2^d/F_2^p$  data [12]. In the top panel shown is  $F_2^d/F_2^p$  shifted by 0, 0.15, ..., 1.5 for  $x = 0.675, 0.450, \dots, 0.0015$ , respectively, as a function of the photon virtuality. The short-dashed line shows a 2 % uncertainty of the ratio as discussed in the text. In the bottom-left panel shown is an up-down asymmetry of the NMC data with respect to the used parametrization and in the bottom-right panel a fraction of the same data points in the parametrization uncertainty band.

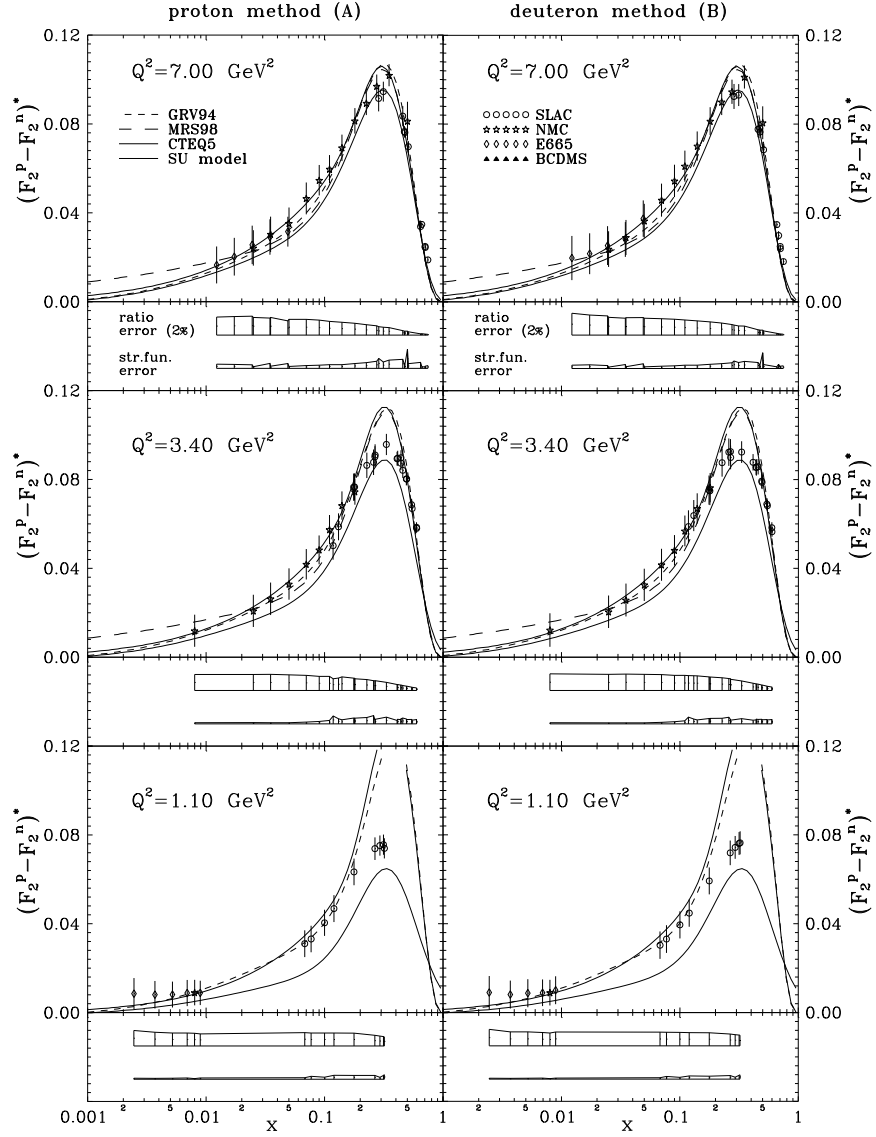


Figure 2:  $D_G(x)$  for some selected  $Q^2$  values obtained with the method A (lhs) and the method B (rhs). Below each distribution shown separately is an error band due to the uncertainty of the  $F_2^n/F_2^p$  ratio and an error band due to the uncertainty of  $F_2^p$  (lhs) or  $F_2^d$  (rhs). The short-dashed line corresponds to the GRV94 NLO [1] PDF's, the long-dashed line to the MRST98 NLO [2] PDF's and the solid to the CTEQ5 NLO [3] PDF's. The thick solid line represents a prediction of our model [1] with details as described in the text.

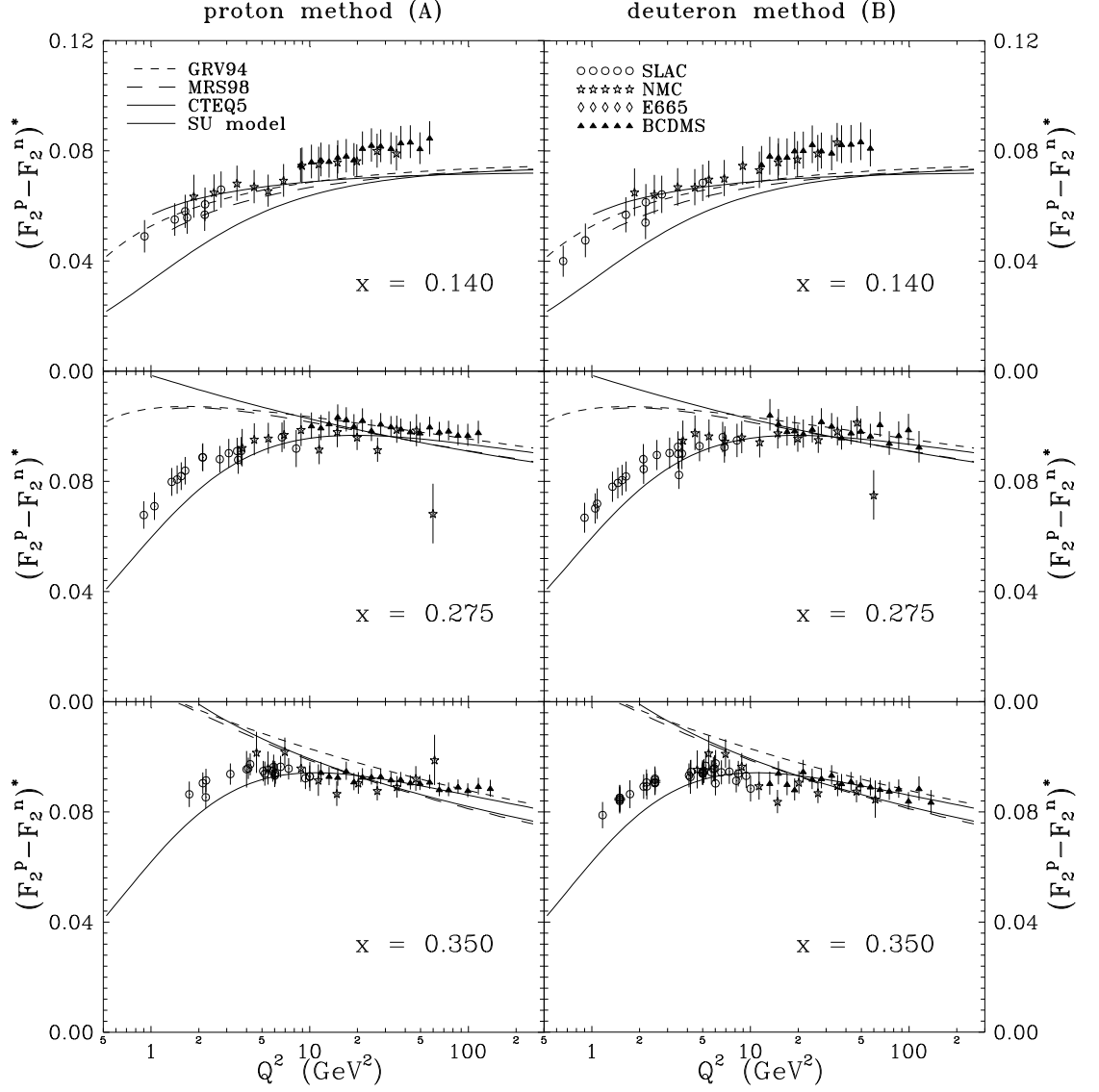


Figure 3:  $D_G(Q^2)$  for some selected values of  $x$ . The meaning of the curves is the same as in Fig.2.